Chapter 5

An Application of Wavelets to Finance: The Three-Factor Fama/French Model

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Additional information is available at the end of the chapter

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Abstract

We use multi-scale analysis and a rolling 250-day window to estimate a widely used standard for empirical asset pricing. The asset pricing model employed is the Fama-French three-factor model. The model is estimated using stock returns for 49 industry stocks of US industry portfolios for the period from July 1969 to September 2017. The rolling window estimation approach allows us to capture the behavior of an investor who periodically reallocates his portfolio. Employing periodic estimates of expected return, we implement a set of long/short investment strategies based on the standard Fama-French three-factor model, and scale versions of the model. We find that during recessions, the higher scale long/short strategies tend to outperform the standard approach. Our results suggest distinct risk dynamics at specific horizons during recessions. We conclude that the information content of the economic phenomena that generate the three-factor model does not follow strict periodicity during recessions, making the wavelet approach more suitable for portfolio managers who must be prepared to rebalance portfolios during official downturns.

Keywords: wavelets, portfolio returns, investment horizon

1. Introduction

The Holy Grail of finance is an empirical asset pricing model that explains stock returns. Most models fall under the risk/return umbrella where risk is positively related to return. There are two basic models in empirical asset pricing, the standard Capital Asset Pricing Model, CAPM [15, 17, 21] and the Fama/French three-factor model, FF3 [5]. The basic idea behind the CAPM is that market movements matter a lot for capturing the relationship between risk and return. The systematic risk measure, beta, is an estimate of the sensitivity of a security or portfolio's

returns to market movements. In the risk/return world, the CAPM is considered a one-factor model in that a single factor, the market return, does all the heavy lifting. The model specification is as follows:

$$r_{it} - rf_t = \alpha_i + \beta_i^* \left(rm_t - rf_t\right) + e_t \tag{1}$$

where r_{it} = return of firm i at time t, rm_t = market return at time t, and rf_t = risk free rate at time t. The slope term, β_{it}, estimates systematic risk. The intercept, α_i, measures abnormal returns, or returns not explained by market exposure of the security or portfolio. In the context of the CAPM, α_i is expected to be zero since only non-diversifiable, also referred to as systematic or market risk, represents the risk that matters for explaining returns.

While the CAPM remains a cornerstone of financial theory, numerous empirical studies have called into question the ability of the CAPM to explain the cross-section of expected stock returns (see for instance, [3]). Several studies have used wavelets to examine the CAPM across scale. Gencay [7] first proposed the use of wavelets to estimate systematic risk in the Capital Asset Pricing Model. They estimate the beta of each stock annually for 6 wavelet scales using daily returns for the period January 1973 to November 2000 for stocks that were in the S&P 500. They find a positive relationship between portfolio returns and beta. Gencay et al. [8] extend their 2003 study by including stocks from the Germany and UK. They find that scale matters in other markets in that the relationship between portfolio returns and beta becomes stronger at high scales. Fernandez [6] applies wavelet analysis to a model of the international CAPM using a data set that consists of daily aggregate equity returns for seven emerging markets for the period 1990–2004.[1] The ICAPM[2] was estimated at 6 scales (2–128 day dynamics). Fernandez finds that market sensitivities are generally greatest at the higher scales of 5 and 6. In addition, the R^2 peaked at scales 5 and 6. She concludes that the ICAPM does its best at capturing the relationship between risk and return at the medium scale or long-term scale that for their data set is 32–128 days. An important takeaway from research employing wavelet measures of beta is that when the environment is distinguished by slowly changing features, or low frequency events the CAPMs' applicability in terms of providing a measure of systematic risk improves when using wavelets. This is consistent with the findings of Rua and Nunes [20] that employs wavelet methodology and provides evidence that market risk varies across time and over frequencies.[3]

The adage the proof of the pudding is in the eating is of particular relevance for empirical asset pricing models. Practitioners want to know if they employ a specific empirical asset pricing model will their investors benefit? The fierce competition to develop a winning model continues among various market players, especially hedge funds [2]. The prescription to basically accept that markets are efficient and form a portfolio that passively tracks the market has contributed to the growth of index investing, but has not slowed the search for a better model.

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[1] Brazil, Chile, Mexico, Indonesia, South Korea, Malaysia, and Thailand.

[2] ICAPM for two countries $E(r_i - r) = \beta_1 \mathrm{cov}(r_i, r_w) + \beta_2 \mathrm{cov}(r_i, s)$, where r_i = returns for domestic asset, r_w = returns for world portfolio, s is the percent change in the exchange rate for domestic and foreign currency.

[3] Their application is to Emerging Markets.

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The idea of basically finding other factors besides the market that explain equity returns has generated many different versions of factor models. One that has gained widespread acceptance is the Fama and French three-factor model (FF3). The general consensus is that the FF3 has greater explanatory power than the CAPM. The Fama-French model adds to the explanatory power of the standard CAPM by including two additional factors, firm size and the book-to-market ratio. Both factors were found in previous research to matter for explaining equity returns. That small firms outperform large cap firms is found in Banz [1], while Barr Rosenberg, Kenneth Reid, and Ronald Lanstein [19] find a positive relationship between average stock returns and book-to-market ratio. Low B/M firms are considered "value stocks" while high B/M are "growth stocks." There is strong consensus around the idea that smaller cap firms are riskier and therefore, generating greater returns beyond what would be expected from simple market beta exposure is a widely accepted explanation for the size factor. There is less agreement for an explanation of the value premium, but one is rooted in behavior where basically relatively cheap stocks outperform relatively expensive stocks because optimism and pessimism persist among investors. Investors bid up growth stocks leading to future under performance, and keep down value stocks leading to future over-performance. Both size and B/M factors are added to their model as factors that account for returns, along with the market factor as found in the CAPM. The FF3 model is specified as follows:

$$r_{it} - rf_t = \alpha_i + \beta_i^* \left(rm_t - rf_t\right) beta_{2i}{}^* SMB_t + \beta_{3i}{}^* HML_t + e_t \tag{2}$$

where SMB_t and HML_t are the size and book-to-market factors, respectively. The book-to-market ratio is intended to capture the difference between value and growth stocks in the sense that the book-to-market ratio is high for value stocks and low for growth stocks.[4]

Several studies have examined the Fama-French 3-factor model at the scale level. Kim and In [10–14] apply wavelets to the Fama-French 3-factor model using monthly data from 1964 to 2004 for 12 industry portfolios. They find that the market variable plays an important role in explaining stock returns across all scales. In addition, they find that the estimated coefficients for the SMB and the HML are significant in specific time scales, depending on the industry. Trimtech et al. [22] apply wavelet analysis to the Fama-French model to study monthly returns for the French stock market for the period 1985. They find that the r-square of the medium and high scale versions of the Fama-French model exceed that of the standard model. They also find that the risk sensitivity of the factors depends on the time scale with the magnitude and sign of the size and book-to-market factors varying across scale.

We use multi-scale analysis and a rolling 250-day window to estimate the Fama-French 3-factor model of stock returns for 49 industry stocks of US industry portfolios. The data set, which consists of daily observations, covers the period from July 1, 1969 to September 29, 2017. We find through risk-adjusting the portfolios using the FF3 model that there are distinct risk dynamics during recessions. The rolling window estimation approach allows us to capture the behavior of an investor who periodically reallocates his portfolio. Using periodic estimates of expected return we implement a set of out-of-sample long/short investment strategies based on

[4]Book-to-market is defined and total assets less total liabilities.

the standard Fama-French model, and also the scale versions of the model. We find that for the sample as a whole the strategy based on the standard model outperforms each of the scale based strategies. In other words, frequency-based information does not appear to matter for portfolio performance when spanning the entire time period. However, during the majority of recessions, the higher scale long/short strategies tend to outperform the standard approach. The frequency content of information does appear to matter during recessions. We conclude that most recessions reflect a time-varying market regime where scale dynamics matter for portfolio performance. In terms of practioners the results suggest that an avenue for potential improvement in portfolio performance is found by taking scale into consideration when faced with potential recessionary periods.

The remainder of this chapter is organized as follows: Section 2 presents the data and basic statistics. Section 3 describes the methodology. Section 4 presents the empirical findings, and Section 5 follows with our concluding comments.

2. Data discussion

Our analysis uses daily equity returns for 49 value-weighted industry portfolios for the period July 1, 1967 to September 29, 2017. The portfolios, which are made available by Kenneth French at his website,[5] are defined by assigning each NYSE, AMEX, and NASDAQ stock to an industry at the end of June in year t, using Compustat 4 digit SIC codes for the fiscal year ending in calendar year t−1. The industry definitions, along with basic statistics for daily returns, are provided in **Table 1**. The returns, which are shown in excess of the risk free rate, range from a low of 0.002% for Real Estate to a high of 0.0522% for Tobacco. The sign of the skewness varies across industries, but the returns for all industries are leptokurtotic.

The period of analysis cover five recessions, which are listed in **Table 2**. Our analysis of the performance of the long/short portfolios across scale focuses on these five recessions.

Excess market returns (Mkt), the risk free rate (RF), and the 2 Fama-French factors (SMB and HML) are also from Kenneth French's website. Excess market returns include all NYSE, AMEX, and NASDAQ firms. The risk free rate is the 1-month Treasury bill rate. The two Fama-French factors are constructed using 6 value-weighted portfolios formed on size and book-to-market. The size factor, SMB (small minus big) is the average return on the three small portfolios minus the average return on the three big portfolios. Similarly, HML (high minus low) is the average return on the three value portfolios minus the average return on the three growth portfolios. **Table 3** contains summary statistics for Mkt, RF, SMB, and HML. The average return for the HML portfolio exceeds that of the SMB portfolio. The HMB portfolio has a small negative skew while Mkt and SMB each have a positive skew. The kurtosis for the SMB portfolio is relatively large.

[5] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Sector	Name	Industry	Mean	Std.Dev	Skewness	Kurtosis
Business Equipment	Chips	Electronic equipment	0.0302	1.6051	0.415	6.48
Business Equipment	Hardw	Computers	0.0247	1.6601	0.048	11.34
Business Equipment	Softw	Computer software	0.0271	2.2342	0.023	7.55
Chemicals	Chems	Chemicals	0.0310	1.2503	−0.142	8.77
Consumer Durables	Hshld	Consumer goods	0.0227	1.0970	−0.236	9.76
Consumer Non-Durables	Agric	Agriculture	0.0305	1.4128	0.390	14.57
Consumer Non-Durables	Beer	Beer & liquor	0.0359	1.1521	−0.401	14.72
Consumer Non-Durables	Books	Printing and publishing	0.0219	1.2104	−0.284	10.71
Consumer Non-Durables	Clths	Apparel	0.0266	1.2706	−0.057	6.84
Consumer Non-Durables	Food	Food products	0.0334	0.9178	−0.044	9.93
Consumer Non-Durables	Smoke	Tobacco products	0.0523	1.4031	−0.366	7.11
Consumer Non-Durables	Soda	Cand & soda	0.0350	1.4357	−0.284	10.67
Consumer Non-Durables	Toys	Recreation	0.0163	1.4800	−0.035	18.54
Consumer Non-Durables	Txtls	Textiles	0.0268	1.3612	−0.843	22.88
Energy	Coal	Coal	0.0318	2.4043	−0.187	8.55
Energy	Mines	Non-metallic and metal	0.0274	1.6273	−0.355	9.47
Energy	Oil	Petroleum and natural gas	0.0306	1.3598	−0.126	3.94
Health	Drugs	Pharmaceutical products	0.0341	1.1545	0.205	10.03
Health	Hlth	Healthcare	0.0243	1.5270	0.471	12.22
Health	MedEq	Medical equipment	0.0309	1.1857	0.117	7.20
Manufacturing	Aero	Aircraft	0.0368	1.3506	−0.197	9.90
Manufacturing	Autos	Automobiles and trucks	0.0213	1.4643	−0.269	6.62
Manufacturing	Boxes	Shipping containers	0.0294	1.2786	−0.319	10.36
Manufacturing	ElcEq	Electrical equipment	0.0355	1.3878	−0.380	10.49
Manufacturing	FabPr	Fabricated products	0.0150	1.5073	−0.441	9.15
Manufacturing	Guns	Defense	0.0424	1.3798	0.246	16.64
Manufacturing	LabEq	Measuring and control equip.	0.0290	1.4337	−0.307	10.14
Manufacturing	Mach	Machinery	0.0272	1.3123	−0.122	7.25
Manufacturing	Paper	Business supplies	0.0276	1.1143	−0.291	14.22
Manufacturing	rubbr	Rubber and plastic products	0.0279	1.1525	−0.131	6.20
Manufacturing	Ships	Shipbuilding, railroad equip.	0.0322	1.5089	−0.296	10.80
Manufacturing	Steel	Steel works, etc.	0.0165	1.6334	−0.236	9.17
Money	Banks	Banking	0.0295	1.4384	−0.184	6.51
Money	Fin	Trading	0.0351	1.4694	−0.564	14.32

Sector	Name	Industry	Mean	Std.Dev	Skewness	Kurtosis
Money	Insur	Insurance	0.0313	1.1682	−0.484	11.24
Money	RlEst	Real estate	0.0022	1.5172	−0.355	8.30
Other	BldMt	Construction materials	0.0282	1.2248	−0.306	7.41
Other	BusSv	Business services	0.0242	1.1153	−0.196	10.40
Other	Cnstr	Construction	0.0246	1.5836	−0.175	6.79
Other	Fun	Entertainment	0.0429	1.6688	0.342	20.74
Other	Gold	Precious metals	0.0244	2.3694	−0.018	16.42
Other	Meals	Restaurants, hotels, motels	0.0301	1.2684	0.299	16.98
Other	Other	Almost nothing	0.0030	1.4295	0.226	15.55
Other	Trans	Transportation	0.0273	1.2429	−0.161	11.68
Shops	PerSv	Personal services	0.0089	1.3192	−0.092	13.46
Shops	Rtail	Retail	0.0305	1.1679	−0.438	6.14
Shops	Whlsl	Wholesale	0.0245	1.0612	−0.528	9.46
Telecommunications	Telcm	Communication	0.0266	1.1191	−0.163	13.71
Utilities	Util	Utilities	0.0239	0.8743	0.012	21.07

Table 1. Daily return statistics (%), July 1, 1967 to September 29, 2017.

Period	Duration (mos)
Nov 1973–Mar 1975	16
Jan–July 1980	6
July 1981–Nov 1982	16
July 1990–Mar 1991	8
Mar 2001–Nov 2001	8
Dec 2007–June 2009	18

Table 2. Recessions and duration in data sample.

	MKt	SMB	HML
Mean	0.0253	0.0032	0.0171
Std. Dev.	1.0248	0.5435	0.5217
Skewness	−0.5049	−1.0605	0.3507
Kurtosis	14.8116	23.2372	9.9377

Table 3. Summary statistics for model factors, daily data, July 1, 1967–September 29, 2017.

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Figures 1 and **2** contain the continuous wavelet power plots and time series plots of returns for Mkt, SMB, and HMB, respectively. For all three series the power tends to be highest for periods less than 256 days. Of the three series, HMB has the highest volatility of returns, and it tends to cluster around the recessionary periods. This is particularly true for the last two recessions. The SMB series has the lowest volatility, however, its power also tends to be highest during recessions.

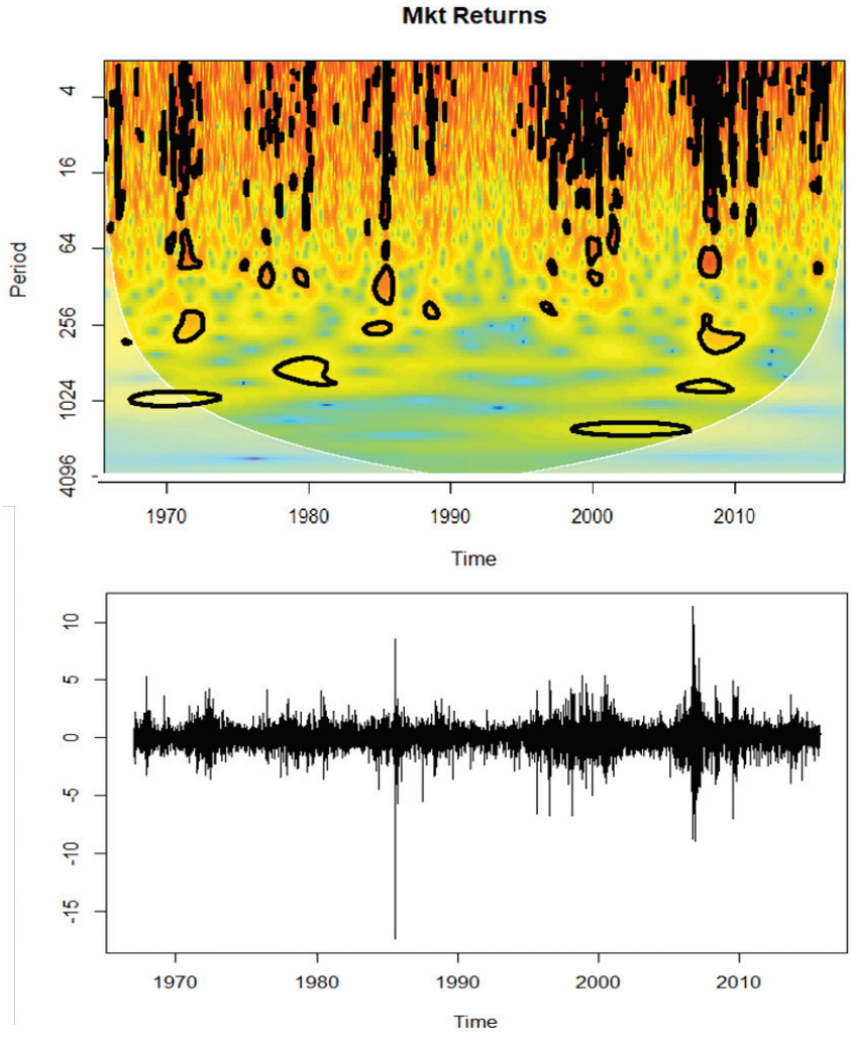

Figure 1. Mkt returns and wavelet power.

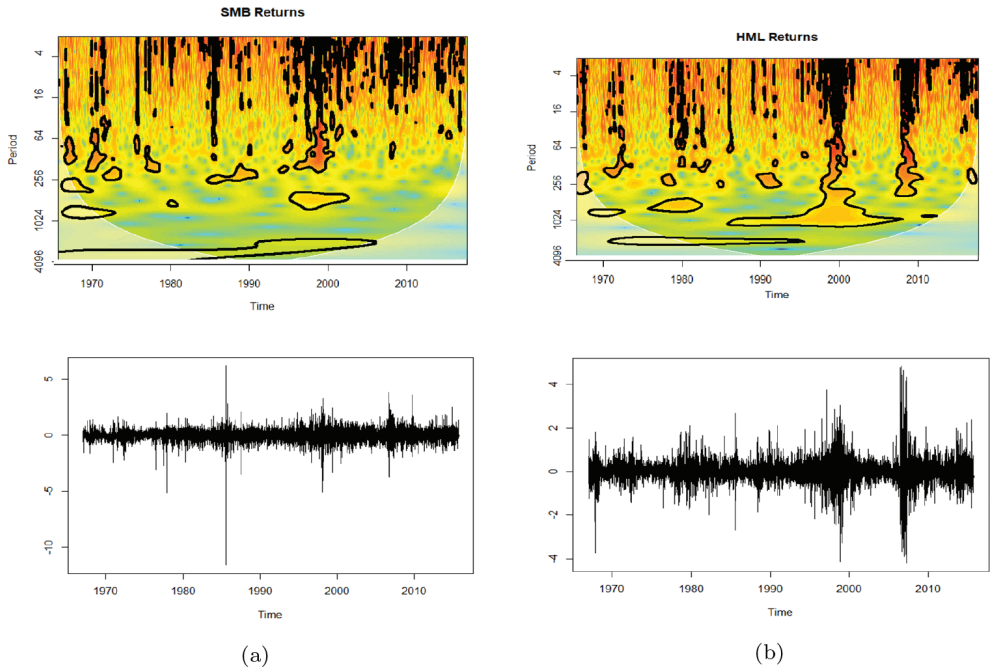

Figure 2. SMB and HML, returns and wavelet power.

3. Methodology

Our analysis of industry returns uses the Maximal Overlap Discrete Wavelet Transform (MODWT). The MODWT is calculated using a pyramid algorithm. Given a data series x_t, a high pass wavelet filter $\tilde{h_1}$, and a low pass scaling filter $\tilde{g_1}$ are applied to obtain wavelet coefficients $\tilde{w}_1$, and scaling coefficients $\tilde{v}_1$. In the second step of the pyramid, the original data series x_t is replaced by $\tilde{v}_1$ which is passed a high pass filter $\tilde{h_2}$ and a low pass filter $\tilde{g_2}$ to obtain wavelet and scaling coefficients, $\tilde{w}_2$, and $\tilde{v}_2$, respectively. This procedure is repeated up to J times where $J = \log_2(N)$. An important feature of the MODWT is that it can be applied to any sample size, while the Discrete Wavelet Transform (DWT) can only be applied to series of size 2^J.[6]

We apply MODWT to each portfolio of industry returns, as well as, the market returns (MKT), the size returns (SMB), and the book-to-market returns. For a filter we choose the Daubechies orthonormal compactly supported wavelet of length L = 8 [4], least asymmetric family. We selected J = 6, common practice in wavelet applications to empirical asset pricing models for providing a good balance in the time and frequency localization. The investment horizons we evaluate cover 2–4 days (J = 1) to 64–128 days (J = 6).

[6]See Chapter 4 of Gencay et al. [9] for additional detail.

3.1. Selecting a filter

In this section, we briefly discuss the process involved in selecting a filter. While our empirical analysis is primarily focused on results using a Daubechies Least Asymmetric filter of length L = 8, LA(8), we also provide results for two other filters to reflect the sensitivity of our results to the filter choice. These two alternative filters are the Daubechies extremal phase filter of length L = 4, DB(4), and the Coiflet filter of length L = 6, C(6).

Percival and Walden [18] point out that in selecting a filter there are two primary considerations, (1) if the filter length is too short it may introduce undesirable anomalies into the results; (2) if the filter is too long more coefficients will be affected by the boundary condition, and there will also be a decrease in the localization of the coefficients. They suggest using the smallest possible filter length that gives reasonable results. They also suggest that if one requires the filter coefficients to be aligned in time, as we do in or analysis, then the LA(8) is generally a good choice. It is not surprising that the LA(8) filter is a very common filter choice in research that applies wavelet methodology to finance.

Figure 3 compares the LA(8) wavelet filter with the two alternative filters used in our analysis. The filter lengths range from 4 to 8. The DB(4) filter has two vanishing moments; the Coiflet(6) has two vanishing moments and is nearly symmetric; the LA(8) has four vanishing moments. The greater the number of vanishing moments the smoother is the scale function.

Since our analysis employs the MODWT, we expect the results to be less sensitive to the filter choice than if we had used a DWT. As discussed in [18] MODWT details and smooths can be generated by averaging circularly shifted DWT details and smooths generated from circularly shifted time series. The averaging smooths out some of the choppiness that is found in DWT MRAs.[7]

3.2. Model specification

The specification of the Fama-French model that we estimated is as follows:

$$\begin{aligned} r_{it}(\lambda_j) - rf_t(\lambda_j) = a_i(\lambda_j) + \beta_i(\lambda_j) * (RM_t(\lambda_j) - RF_t(\lambda_j)) \\ + \beta_{2i}(\lambda_j) * SMB_t(\lambda_j) + \beta_3 i * (\lambda_j) * HML_t(\lambda_j) + e_{it}(\lambda_j) \end{aligned} \tag{3}$$

where $\lambda = 2^{j-1}$, for $j = 1, \ldots, 6$. $r_{it}(\lambda_j) - rf_t(\lambda_j)$ is the excess return for industry portfolio i and time t, and scale j. $RM_t\lambda_j$, $RM_t\lambda_j$, $SMB_t(\lambda_j)$, and $HML_t(\lambda_j)$ are the Fama-French factor for scale, j.

After we disaggregate the series to scale we use a rolling 250-day window to estimate the standard model, and each of the six scale level models. Each time we estimate the models we calculate the expected return for each industry as of the last day of the estimation period.

[7] Percival and Walden provide a comparison of DWT and MODWT smooths for various filters which shows that MODWT MRAs are less sensitive to the filter type than DWT MRAs. See pp. 195–200 in Percival and Walden for a discussion on the practical considerations of the MODWT.

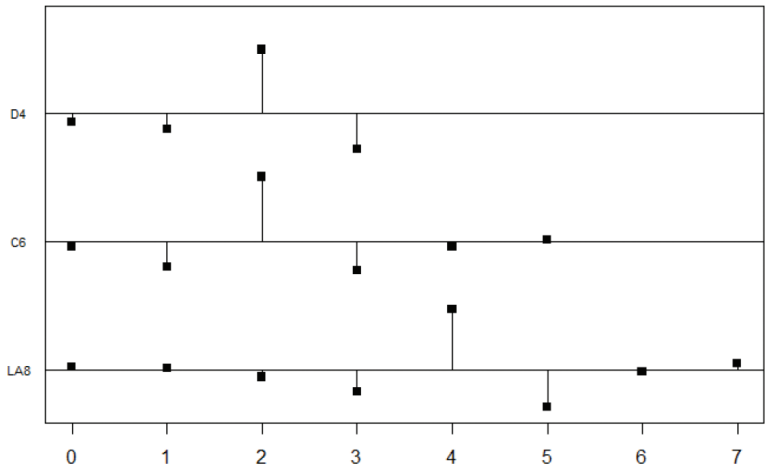

Figure 3. Three wavelet filters—DB(4), C(6), and LA(8).

We then rank the expected returns for that estimation period and assign a decile. The long-short strategy that we employ consists of going long (buying) the top decile, and going short (selling) the bottom decile. This position is held for 20 days. At the end of the 20 days period we re-estimate the models using the previous 250 days and repeat the investment selection process. Since there are 49 industry portfolios, this means that every 20 days we create a portfolio that is long 5 industries and short 5 industries. We calculate the out-of-sample cumulative returns for each 20-day period. We roll this process forward for the entire sample period.

4. Empirical findings

Our discussion of the empirical findings consists of four parts. We begin with a comparison of the parameters for the standard model parameters and the 6 scale models for the LA(8) filter. We discuss both sector averages, and industry results. Next, we examine parameter estimates for the alternative filters, DB(4) and C(6). We then discuss the returns for the long/short strategy at each scale over the entire sample period. Finally, we turn our focus to the performance of the strategies during periods of recession.

4.1. Parameter estimates

4.1.1. LA(8) filter

Table 4 contains sector level averages of the industry 'beta' parameter estimates. The difference between the standard model and the scale models for the industries tends to be modest. This is generally consistent with studies that have used monthly data to evaluate sector returns across scale. For instance using the CAPM, McNevin and Nix [16] found only small differences between the standard beta and wavelet betas for scales 1 and 2.

Sector	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	1.184	1.215	1.162	1.138	1.139	1.149	1.147
Chemicals	1.083	1.049	1.086	1.102	1.136	1.176	1.084
Consumer Durables	0.849	0.874	0.831	0.822	0.764	0.769	0.857
Consumer Non-Durables	0.889	0.887	0.891	0.890	0.897	0.879	0.907
Energy	1.108	1.092	1.140	1.141	1.102	1.189	1.018
Health	0.955	0.963	0.986	0.960	0.945	0.948	0.866
Manufacturing	1.061	1.051	1.046	1.083	1.092	1.110	1.070
Money	1.091	1.062	1.086	1.123	1.133	1.109	1.214
Other	1.015	0.996	1.022	1.051	1.043	1.033	0.982
Shops	1.014	1.020	1.015	1.012	1.031	1.038	0.987
Telecommunications	0.888	0.941	0.881	0.867	0.830	0.849	0.805
Utilities	0.709	0.707	0.713	0.729	0.739	0.708	0.728

Table 4. Average Beta parameter by sector—LA(8).

Table 15 (in Appendix) contains the industry level parameter estimates of the market variable, or the 'betas'. These parameters are averages of the rolling window estimates. There were a total of 597 rolling window regressions. On average, all of the parameter estimates in **Table 15** are significant at the 95% level of confidence. **Table 16** contains the corresponding t-statistics. There is no definitive pattern to the parameters across scale, though they tend to increase with scale.

Table 5 contains average sector parameters for the size variables. The range of parameters for the Business Equipment sector is the greatest, ranging from 0.092 for scale 1 to 0.463 for scale 6. Most of the other sectors do not exhibit a strong pattern across scale. The parameter estimates for utilities change sign across scale. In this case the sector and industry parameters are the same. An examination of **Table 18** indicates that the standard model size parameter is insignificant for the utilities, but the parameters for scales 4–6 are all negative and significant. As shown in **Table 17**, the size parameter at the industry level can vary quite a bit across scale and in comparison to the standard model indicating that in some industries investors require a premium for investing in small firm stocks over longer investment horizons. Some examples include Chips, Software, Mines, Steel, Gold, and Lab. equipment.

Table 6 contains the average sector parameter estimates for the book-to-market factor. Two sectors with notable differences across scale are Chemicals and Energy. The Chemical sector only contains a single industry. **Table 20** shows the t-statistics for the HML parameter at the industry level. On average, for the standard model the HML parameter is not statistically significant. However, it is positive and significant at scales 3–6. **Table 19** contains the industry level parameters for the HML risk factor. As is the case with SMB, the importance of the HML factor across scale varies widely by industry. Notable difference across scale can be seen in Coal, Lab. Equipment, and Construction.

Sector	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	0.175	0.092	0.182	0.242	0.312	0.412	0.463
Chemicals	0.066	0.041	0.090	0.110	0.088	0.029	0.041
Consumer Durables	−0.270	−0.289	−0.290	−0.275	−0.175	−0.132	−0.207
Consumer Non-Durables	0.153	0.166	0.147	0.134	0.135	0.188	0.146
Energy	0.247	0.231	0.248	0.291	0.348	0.275	0.303
Health	0.158	0.186	0.164	0.139	0.114	0.120	0.179
Manufacturing	0.260	0.249	0.242	0.297	0.281	0.276	0.291
Money	0.286	0.304	0.291	0.249	0.216	0.190	0.244
Other	0.362	0.342	0.375	0.382	0.366	0.354	0.408
Shops	0.353	0.369	0.358	0.318	0.268	0.320	0.384
Telecommunications	−0.196	−0.168	−0.223	−0.207	−0.211	−0.249	−0.091
Utilities	−0.031	0.029	−0.018	−0.031	−0.147	−0.231	−0.337

Table 5. Average size parameter by sector—LA(8).

Sector	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	−0.661	−0.640	−0.686	−0.712	−0.672	−0.645	−0.583
Chemicals	0.193	0.145	0.189	0.228	0.270	0.242	0.347
Consumer Durables	−0.231	−0.235	−0.231	−0.222	−0.260	−0.388	−0.227
Consumer Non-Durables	−0.021	−0.013	−0.016	−0.037	−0.042	−0.069	0.028
Energy	0.444	0.355	0.489	0.495	0.554	0.578	0.653
Health	−0.342	−0.268	−0.328	−0.395	−0.394	−0.411	−0.489
Manufacturing	0.188	0.202	0.176	0.197	0.189	0.103	0.152
Money	0.392	0.380	0.380	0.360	0.367	0.394	0.442
Other	0.077	0.070	0.077	0.095	0.074	0.071	0.155
Shops	−0.014	0.025	−0.003	−0.039	−0.035	−0.072	−0.114
Telecommunications	0.253	0.305	0.275	0.226	0.196	0.274	0.097
Utilities	0.418	0.372	0.422	0.435	0.467	0.513	0.395

Table 6. Average book-to-market parameter by sector—LA(8).

4.1.2. Alternative filter parameter estimates: DB(4), C(6) filters

In this section, we provide sector averages of parameter estimates for the Fama-French model based on two alternative filters.[8] **Tables 7** and **8** contain the average sector betas for the DB(4) and C(6) filters, respectively. The sector level averages for the two alternative filters are quite similar. What is important for our analysis is that they are similar to the results for the LA(8)

[8]Industry level parameter estimates and t-statistics for the alternative filters are available from the authors upon request.

Sector	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	1.210	1.162	1.141	1.141	1.143	1.137
Chemicals	1.051	1.086	1.103	1.131	1.164	1.109
Consumer Durables	0.869	0.835	0.824	0.774	0.781	0.849
Consumer Non-Durables	0.887	0.892	0.889	0.896	0.882	0.905
Energy	1.095	1.133	1.139	1.108	1.161	1.053
Health	0.964	0.983	0.961	0.946	0.949	0.879
Manufacturing	1.050	1.050	1.079	1.091	1.107	1.085
Money	1.063	1.086	1.119	1.130	1.116	1.201
Other	0.996	1.021	1.048	1.043	1.026	1.000
Shops	1.018	1.016	1.012	1.027	1.034	1.002
Telecommunications	0.937	0.883	0.863	0.835	0.848	0.814
Utilities	0.708	0.713	0.727	0.735	0.701	0.714

Table 7. Average Beta parameter by sector—DB(4).

Sector	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	1.211	1.162	1.142	1.141	1.146	1.139
Chemicals	1.051	1.086	1.103	1.132	1.168	1.105
Consumer Durables	0.870	0.835	0.825	0.773	0.780	0.848
Consumer Non-Durables	0.887	0.891	0.889	0.896	0.882	0.907
Energy	1.095	1.133	1.141	1.109	1.162	1.043
Health	0.964	0.983	0.961	0.947	0.948	0.878
Manufacturing	1.050	1.049	1.079	1.091	1.108	1.082
Money	1.063	1.086	1.119	1.131	1.117	1.202
Other	0.996	1.021	1.048	1.043	1.028	1.000
Shops	1.018	1.016	1.012	1.028	1.035	1.002
Telecommunications	0.937	0.883	0.864	0.835	0.847	0.814
Utilities	0.708	0.713	0.727	0.735	0.701	0.716

Table 8. Average Beta parameter by sector—C(6).

filter (**Table 4**). **Tables 9** and **10** contain the sector parameter estimates for the firm size variable for the DB(4) and C(6) filters, respectively. These parameter estimates are also similar across filters. **Tables 11** and **12** show the parameters for the book-to-market variable for the alternative filters. In summary, there is very little difference in parameter estimates across the different filters.

Our comparison of paramater estimates across filters provides support that our parameter estimates based on the MODWT are not over sensitive to the choice of a filter. The remainder of

Sector	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	0.094	0.176	0.241	0.302	0.406	0.447
Chemicals	0.043	0.083	0.108	0.089	0.026	0.042
Consumer Durables	−0.290	−0.288	−0.274	−0.187	−0.143	−0.211
Consumer Non-Durables	0.165	0.148	0.134	0.135	0.175	0.131
Energy	0.234	0.245	0.294	0.334	0.286	0.309
Health	0.185	0.164	0.140	0.121	0.122	0.147
Manufacturing	0.249	0.244	0.292	0.284	0.275	0.292
Money	0.303	0.291	0.253	0.221	0.190	0.232
Other	0.343	0.370	0.380	0.365	0.360	0.395
Shops	0.369	0.355	0.318	0.278	0.318	0.371
Telecommunications	−0.172	−0.215	−0.214	−0.212	−0.221	−0.101
Utilities	0.027	−0.015	−0.039	−0.138	−0.215	−0.315

Table 9. Average size parameter by sector—DB(4).

Sector	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	0.094	0.176	0.242	0.301	0.404	0.446
Chemicals	0.043	0.083	0.108	0.089	0.028	0.043
Consumer Durables	−0.290	−0.287	−0.274	−0.186	−0.142	−0.209
Consumer Non-Durables	0.165	0.148	0.136	0.136	0.178	0.135
Energy	0.233	0.245	0.292	0.332	0.279	0.305
Health	0.185	0.164	0.140	0.120	0.125	0.156
Manufacturing	0.249	0.244	0.291	0.284	0.274	0.296
Money	0.303	0.291	0.255	0.222	0.188	0.231
Other	0.343	0.371	0.381	0.366	0.357	0.397
Shops	0.369	0.355	0.319	0.280	0.319	0.375
Telecommunications	−0.172	−0.215	−0.213	−0.213	−0.223	−0.105
Utilities	0.027	−0.015	−0.037	−0.139	−0.217	−0.319

Table 10. Average size parameter by sector—C(6).

the chapter focuses on the results for the LA(8) filter—a filter that is widely used in finance research employing wavelet methodology.

4.2. Long-short strategy

In this section, we review the results of the long/short strategies applied over time. We begin by examining the average statistics for the out-of-sample results for both the standard Fama-French model and each of the scales. **Table 13** presents a summary of the results.

Sector	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	−0.640	−0.685	−0.700	−0.682	−0.634	−0.611
Chemicals	0.147	0.190	0.227	0.260	0.252	0.348
Consumer Durables	−0.236	−0.226	−0.221	−0.261	−0.356	−0.225
Consumer Non-Durables	−0.014	−0.017	−0.036	−0.042	−0.061	0.022
Energy	0.366	0.475	0.500	0.552	0.570	0.624
Health	−0.271	−0.327	−0.395	−0.388	−0.419	−0.465
Manufacturing	0.201	0.181	0.195	0.184	0.117	0.160
Money	0.381	0.382	0.364	0.375	0.393	0.447
Other	0.070	0.078	0.093	0.075	0.076	0.146
Shops	0.020	−0.002	−0.036	−0.038	−0.070	−0.085
Telecommunications	0.302	0.273	0.221	0.206	0.252	0.105
Utilities	0.376	0.420	0.435	0.468	0.500	0.385

Table 11. Average book-to-market parameter by sector—DB(4).

Sector	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	−0.640	−0.684	−0.701	−0.682	−0.637	−0.609
Chemicals	0.147	0.190	0.227	0.261	0.256	0.349
Consumer Durables	−0.236	−0.227	−0.220	−0.262	−0.358	−0.224
Consumer Non-Durables	−0.014	−0.017	−0.036	−0.042	−0.061	0.021
Energy	0.365	0.476	0.498	0.553	0.573	0.636
Health	−0.271	−0.328	−0.392	−0.389	−0.419	−0.464
Manufacturing	0.201	0.180	0.195	0.183	0.116	0.160
Money	0.381	0.381	0.363	0.373	0.391	0.442
Other	0.070	0.078	0.093	0.075	0.076	0.148
Shops	0.021	−0.002	−0.037	−0.040	−0.072	−0.090
Telecommunications	0.302	0.273	0.220	0.207	0.258	0.107
Utilities	0.376	0.420	0.435	0.469	0.502	0.391

Table 12. Average book-to-market parameter by sector—DB(4).

On average the cumulative 20-day return for the standard model (2.47%) exceeds all of the scale models. The scale 4 model has the second highest average cumulative returns (1.71%). The standard deviations are quite similar for all 7 models. The minimum and maximum cumulative returns are both quite high for all 7 models. This reflects the fact that there are only 10 positions in the out-of-sample portfolio at any point in time. It may also reflect the fact that the positions in the portfolio have equal weights (in absolute value). Finally, the Sharpe ratio for each of the models, even the standard model, is close to zero.

	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Mean	2.47	−0.60	0.40	−0.54	1.71	−0.70	0.85
Std. dev	29.75	28.28	28.50	27.24	26.92	25.81	23.45
Skewness	−0.34	0.18	−0.09	−0.07	−0.09	−0.29	−0.20
Kurtosis	2.48	1.60	1.73	1.68	2.50	1.62	0.90
Minimum	−166.32	−112.08	−110.44	−117.63	−109.45	−127.75	−97.01
Maximum	106.20	115.68	105.22	100.11	125.54	87.21	74.49
Median	3.82	−0.35	1.21	−0.03	−0.07	0.35	1.08
Sharpe Ratio	0.08	−0.02	0.01	−0.02	0.06	−0.03	0.04

Table 13. Average 20-day cumulative returns for long-short strategy—LA(8).

4.3. Strategy performance during economic recessions

While the scale level model does not seem to improve the long/short strategy overall, an examination of the returns during recessions tells a different story. As shown in **Table 14** and **Figures 4–6** for four of six recessions the returns at scale level exceed those using the standard model.

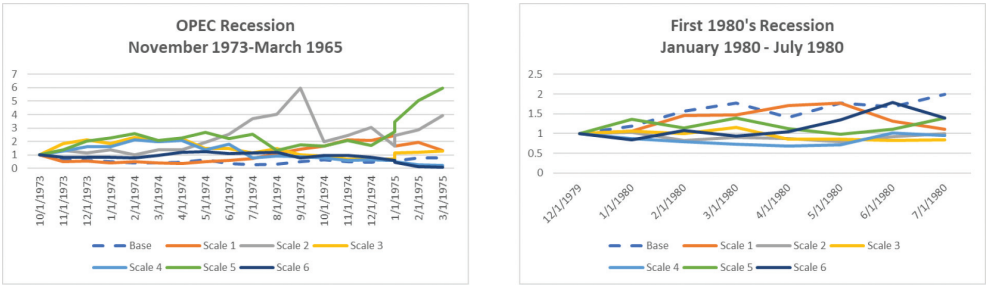

Figure 4. Out-of-sample returns—long-short strategy LA(8).

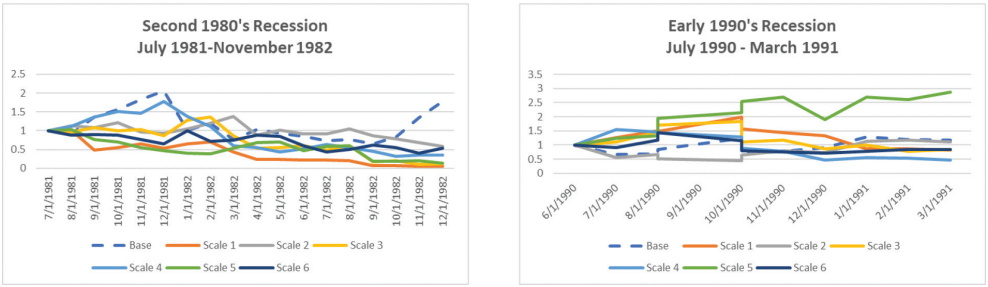

Figure 5. Out-of-sample returns—long-short strategy LA(8).

Recession	Base	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Nov 1973–Mar 1975	0.78	1.35	3.94	1.28	0.22	5.97	0.10
Jan–July 1980	2.0	1.11	1.00	0.84	0.96	1.39	1.39
July 1981–Nov 1982	1.53	0.07	0.55	0.09	0.24	0.10	0.54
July 1990–Mar 1991	1.17	0.81	1.11	0.84	0.46	2.88	0.84
Mar 2001–Nov 2001	1.99	0.30	12.34	0.05	0.08	0.03	5.35
Dec 2007–June 2009	0.00	0.01	0.00	−0.41	0.11	0.00	2.35

Table 14. Cumulative out-of-sample returns—LA(8).

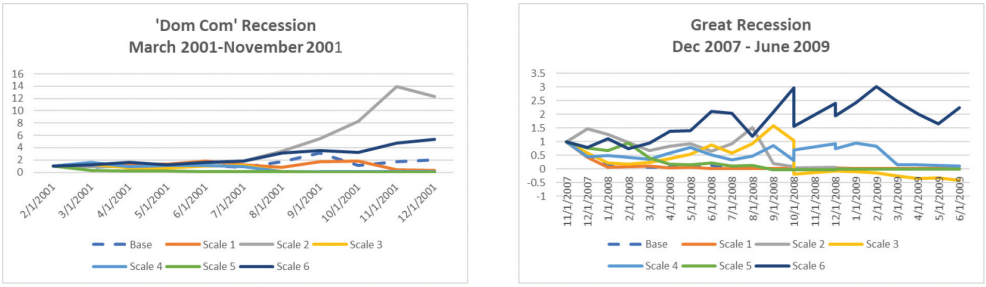

Figure 6. Out-of-sample returns—long-short strategy LA(8).

In particular, the deep recession of the 1970s, as well as, the more recent financial crisis, illustrates how scale effects matter for designing portfolios that maximize returns (**Figures 4–6** and **Table 14**).

5. Conclusion

The focus of this chapter is on whether adding wavelet methodology to the FF3 model is really "worth it." We attempt to show why it makes sense to add this methodology to the empirical asset pricing toolkit, and ultimately why practitioners should also consider including wavelet methodology in the mix of empirical asset pricing techniques used to provide advice and select portfolios for clients. The most fundamental reason for answering in the affirmative regarding whether wavelet methodology should have a seat at the table of empirical asset pricing models is that when an identified risk "signal" shows different behavior at different time periods, wavelet analysis, capable of decomposing data into several time scales, allows the researcher an opportunity to investigate the behavior of the risk factor/signal over various time scales. The exploration is richer because it allows windows to vary. Of course, allowing for risk measures that vary over time and across frequencies is not the same as finding that it will always matter for the results when compared to a standard approach devoid of such possibilities. Consistent with other research employing scale versions of the FF3 model, we find

industry-specific effects on size and HML factors that are absent using the standard model. The large-scale versus fine-scale information distinction that the scale version of the FF3 model is capable of capturing is found significant for portfolio performance during the majority of recessions included in our data. Finding that the wavelet-based version of the FF3 model produces better portfolio outcomes is of importance to practioners, as well as, researchers. Our main conclusion based on the inter-temporal behavior of financial characteristics estimated with the FF3 model is that risk measures that vary over time and across frequencies are needed to capture the risk dynamics associated with most downturns. The importance of scale effects during periods defined as recessions leads us to conclude that the distinct risk dynamics during recessions are better captured with a methodology that allows for scale effects, providing yet another reason why wavelet methodology is a worthwhile tool that belongs in the methodological toolbox of practitioners in finance.

A. Appendix

See **Tables 15–20**.

Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	Chips	1.190	1.192	1.160	1.145	1.183	1.193	1.309
Business Equipment	Hardw	1.045	1.045	0.988	0.974	1.031	1.054	1.017
Business Equipment	Softw	1.318	1.408	1.337	1.295	1.204	1.200	1.114
Chemicals	Chems	1.083	1.049	1.086	1.102	1.136	1.176	1.084
Consumer Durables	Hshld	0.849	0.874	0.831	0.822	0.764	0.769	0.857
Consumer Non-Durables	Agric	0.863	0.894	0.844	0.851	0.894	0.850	0.826
Consumer Non-Durables	Beer	0.749	0.749	0.762	0.748	0.689	0.734	0.827
Consumer Non-Durables	Books	0.913	0.872	0.916	0.929	0.932	1.020	1.071
Consumer Non-Durables	Clths	1.041	1.023	1.041	1.080	1.064	1.023	0.982
Consumer Non-Durables	Food	0.732	0.759	0.741	0.724	0.701	0.722	0.718
Consumer Non-Durables	Smoke	0.815	0.814	0.818	0.828	0.859	0.756	0.817
Consumer Non-Durables	Soda	0.810	0.846	0.798	0.741	0.775	0.742	0.892
Consumer Non-Durables	Toys	1.072	1.050	1.066	1.063	1.138	1.053	1.017
Consumer Non-Durables	Txtls	1.011	0.977	1.031	1.047	1.018	1.012	1.011
Energy	Coal	1.211	1.212	1.266	1.238	1.127	1.269	1.091
Energy	Mines	1.108	1.064	1.138	1.172	1.192	1.229	1.060
Energy	Oil	1.004	1.001	1.016	1.012	0.988	1.071	0.902

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Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Health	Drugs	0.841	0.850	0.874	0.837	0.815	0.867	0.793
Health	Hlth	1.086	1.090	1.112	1.093	1.108	1.073	1.030
Health	MedEq	0.938	0.949	0.972	0.949	0.911	0.903	0.776
Manufacturing	Aero	1.128	1.120	1.137	1.159	1.150	1.146	1.129
Manufacturing	Autos	1.216	1.208	1.199	1.220	1.269	1.193	1.151
Manufacturing	Boxes	0.962	0.963	0.960	1.005	0.960	1.000	0.986
Manufacturing	ElcEq	1.052	1.046	1.024	1.078	1.063	1.091	1.030
Manufacturing	FabPr	1.043	1.045	1.014	1.073	1.145	1.059	1.080
Manufacturing	Guns	0.868	0.883	0.816	0.848	0.863	0.900	0.858
Manufacturing	LabEq	1.113	1.096	1.107	1.105	1.126	1.125	1.108
Manufacturing	Mach	1.146	1.116	1.145	1.170	1.189	1.217	1.094
Manufacturing	Paper	0.983	0.971	0.982	0.992	1.021	1.059	1.100
Manufacturing	Rubbr	0.938	0.927	0.920	0.967	0.958	0.970	0.979
Manufacturing	Ships	0.973	0.949	0.932	1.014	1.018	1.167	1.045
Manufacturing	Steel	1.309	1.284	1.316	1.362	1.348	1.392	1.275
Money	Banks	1.149	1.102	1.157	1.177	1.194	1.163	1.269
Money	Fin	1.179	1.135	1.174	1.227	1.242	1.187	1.337
Money	Insur	1.015	0.990	1.024	1.036	1.061	1.057	1.128
Money	RlEst	1.021	1.021	0.988	1.050	1.035	1.028	1.122
Other	BldMt	1.056	1.024	1.066	1.100	1.088	1.057	1.085
Other	BusSv	1.028	1.021	1.033	1.051	1.064	1.075	1.030
Other	Cnstr	1.277	1.239	1.313	1.335	1.343	1.324	1.249
Other	Fun	1.155	1.162	1.182	1.141	1.131	1.161	1.206
Other	Gold	0.418	0.337	0.381	0.560	0.551	0.508	0.303
Other	Meals	1.006	1.011	1.002	0.990	1.010	0.967	0.926
Other	Other	1.030	1.022	1.045	1.080	0.979	1.019	1.038
Other	Trans	1.149	1.151	1.151	1.156	1.177	1.155	1.016
Shops	PerSv	1.045	1.057	1.039	1.029	1.073	1.109	1.080
Shops	Rtail	1.015	1.016	1.038	1.008	1.022	0.969	0.944
Shops	Whlsl	0.982	0.988	0.968	1.000	0.998	1.037	0.936
Telecommunications	Telcm	0.888	0.941	0.881	0.867	0.830	0.849	0.805
Utilities	Util	0.709	0.707	0.713	0.729	0.739	0.708	0.728

Table 15. Average betas by industry.

Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	Chips	20.058	18.676	18.987	20.548	22.601	24.972	31.644
Business Equipment	Hardw	14.283	13.223	13.377	14.701	16.357	17.872	20.137
Business Equipment	Softw	16.634	16.472	15.829	16.850	17.844	18.151	22.771
Chemicals	Chems	21.925	19.738	21.799	23.713	25.057	28.151	27.495
Consumer Durables	Hshld	17.380	16.028	16.498	18.202	19.852	22.056	27.869
Consumer Non-Durables	Agric	8.821	8.099	8.490	9.475	10.377	11.386	14.378
Consumer Non-Durables	Beer	10.932	9.917	10.975	11.545	11.508	13.507	17.300
Consumer Non-Durables	Books	16.103	13.937	15.865	18.200	19.563	22.655	27.925
Consumer Non-Durables	Clths	16.411	14.663	16.097	18.237	19.715	21.163	21.696
Consumer Non-Durables	Food	16.342	15.236	16.662	18.015	18.750	19.287	18.581
Consumer Non-Durables	Smoke	9.323	8.550	9.024	9.906	11.156	11.715	12.592
Consumer Non-Durables	Soda	8.714	8.010	8.347	9.161	10.212	10.908	14.548
Consumer Non-Durables	Toys	12.408	10.779	12.248	13.785	16.075	16.381	16.935
Consumer Non-Durables	Txtls	14.762	12.818	14.965	16.364	17.847	19.035	19.842
Energy	Coal	8.114	7.372	8.192	8.795	8.884	10.583	10.666
Energy	Mines	12.208	10.836	12.522	13.965	13.910	15.223	15.581
Energy	Oil	15.789	15.538	15.541	16.156	16.021	18.747	16.642
Health	Drugs	16.934	16.820	16.901	17.337	17.415	18.129	18.117
Health	Hlth	12.257	11.240	12.195	13.315	14.145	14.559	15.944
Health	MedEq	15.091	13.963	15.091	16.873	16.795	18.946	17.974
Manufacturing	Aero	16.333	14.903	15.914	18.339	19.158	20.647	22.643
Manufacturing	Autos	17.121	15.669	16.714	18.524	20.197	21.464	22.892
Manufacturing	Boxes	13.697	12.244	13.521	15.476	15.640	20.005	22.820
Manufacturing	ElcEq	17.729	16.077	17.329	19.733	20.469	23.810	25.480
Manufacturing	FabPr	11.949	10.657	11.535	13.547	15.453	16.114	18.031
Manufacturing	Guns	9.236	8.155	8.858	9.972	11.309	13.892	14.047
Manufacturing	LabEq	19.115	17.755	18.607	20.774	21.530	23.382	27.771
Manufacturing	Mach	25.353	23.134	25.554	27.713	28.412	31.287	30.476
Manufacturing	Paper	19.577	18.314	19.622	20.704	21.706	25.180	26.474
Manufacturing	Rubbr	15.763	14.015	15.287	18.207	19.327	22.154	26.683
Manufacturing	Ships	10.150	9.042	9.782	11.332	12.203	16.396	17.330
Manufacturing	Steel	18.723	17.538	18.439	20.513	20.903	22.971	23.042
Money	Banks	23.902	23.097	23.248	24.515	24.899	27.025	32.536
Money	Fin	28.065	24.606	27.841	32.581	34.215	35.430	42.598
Money	Insur	22.791	21.579	22.820	24.652	25.690	25.437	27.779
Money	RlEst	13.792	12.192	13.428	15.351	16.482	18.424	23.825

Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Other	BldMt	23.160	20.729	23.102	25.692	27.509	28.187	30.079
Other	BusSv	29.792	26.771	29.044	34.174	36.579	43.397	49.071
Other	Cnstr	16.446	14.352	16.451	18.835	20.246	22.905	22.818
Other	Fun	12.920	11.579	13.058	14.071	15.282	16.892	20.068
Other	Gold	2.172	1.752	1.859	2.948	3.147	2.995	2.716
Other	Meals	15.406	14.145	14.902	16.216	18.574	20.730	21.741
Other	Other	19.085	17.702	18.328	20.316	21.834	24.979	28.458
Other	Trans	20.572	19.167	19.705	22.067	23.519	26.414	23.600
Shops	PerSv	14.241	12.781	13.989	15.642	17.178	19.807	22.745
Shops	Rtail	21.002	20.071	20.629	21.615	24.246	24.585	25.547
Shops	Whlsl	24.826	22.538	24.086	27.632	30.981	36.594	37.185
Telecommunications	Telcm	17.784	17.634	16.979	18.254	18.723	20.464	21.987
Utilities	Util	17.385	17.873	18.222	18.486	17.156	17.952	20.056

Table 16. Average t-statistics for the Mkt risk parameter.

Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	Chips	0.1998	0.1560	0.1574	0.2144	0.2877	0.3544	0.3876
Business Equipment	Hardw	−0.2445	−0.4060	−0.2962	−0.1405	0.0260	0.1262	0.1714
Business Equipment	Softw	0.5691	0.5265	0.6843	0.6507	0.6234	0.7560	0.8310
Chemicals	Chems	0.0664	0.0408	0.0903	0.1104	0.0882	0.0286	0.0412
Consumer Durables	Hshld	−0.2695	−0.2885	−0.2902	−0.2755	−0.1750	−0.1320	−0.2072
Consumer Non-Durables	Agric	0.3758	0.4000	0.3346	0.4365	0.3317	0.4373	0.4856
Consumer Non-Durables	Beer	−0.1894	−0.1742	−0.1779	−0.2261	−0.2481	−0.1270	−0.1546
Consumer Non-Durables	Books	0.3068	0.3146	0.3086	0.2664	0.2520	0.2068	0.1749
Consumer Non-Durables	Clths	0.4596	0.4600	0.4668	0.4470	0.3915	0.4289	0.4416
Consumer Non-Durables	Food	−0.0900	−0.0347	−0.0801	−0.1009	−0.1251	−0.1603	−0.2084
Consumer Non-Durables	Smoke	−0.2571	−0.2403	−0.2177	−0.3084	−0.2380	−0.2543	−0.2380
Consumer Non-Durables	Soda	−0.1360	−0.0748	−0.1717	−0.2282	−0.2255	0.0180	−0.0552
Consumer Non-Durables	Toys	0.2947	0.2337	0.2728	0.3009	0.3984	0.6079	0.3526
Consumer Non-Durables	Txtls	0.6164	0.6125	0.5859	0.6208	0.6822	0.5305	0.5190
Energy	Coal	0.5192	0.5225	0.4946	0.5778	0.7556	0.5183	0.2613
Energy	Mines	0.4235	0.4023	0.4291	0.4710	0.3921	0.4007	0.7195
Energy	Oil	−0.2021	−0.2319	−0.1798	−0.1753	−0.1043	−0.0953	−0.0733
Health	Drugs	−0.2174	−0.1567	−0.2341	−0.2440	−0.2329	−0.2875	−0.3023

Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Health	Hlth	0.5992	0.6107	0.6146	0.6126	0.4997	0.5490	0.6607
Health	MedEq	0.0919	0.1046	0.1111	0.0494	0.0739	0.0974	0.1784
Manufacturing	Aero	0.1087	0.0759	0.1182	0.1603	0.1775	0.1178	0.1247
Manufacturing	Autos	−0.0033	−0.1007	−0.0018	0.0975	0.1487	0.1189	0.2935
Manufacturing	Boxes	0.1144	0.1614	0.1109	0.1248	0.1137	0.0077	−0.1318
Manufacturing	ElcEq	0.0835	0.0799	0.0182	0.1073	0.1636	0.1478	0.1187
Manufacturing	FabPr	0.7137	0.7314	0.7014	0.7544	0.7237	0.6921	0.6956
Manufacturing	Guns	0.0159	−0.0055	−0.0323	0.0950	0.0378	0.0723	0.0229
Manufacturing	LabEq	0.3092	0.2681	0.2857	0.2968	0.3211	0.4689	0.5188
Manufacturing	Mach	0.3527	0.3419	0.3153	0.3799	0.4194	0.4127	0.3551
Manufacturing	Paper	0.1091	0.1384	0.1146	0.1011	0.0161	0.0108	−0.0158
Manufacturing	Rubbr	0.5363	0.5332	0.4960	0.5663	0.5263	0.5307	0.3955
Manufacturing	Ships	0.3484	0.3504	0.3106	0.4258	0.2808	0.2984	0.3986
Manufacturing	Steel	0.4330	0.4086	0.4614	0.4548	0.4483	0.4327	0.7188
Money	Banks	0.0651	0.0740	0.0425	0.0219	0.0581	−0.0553	0.0099
Money	Fin	0.2349	0.2459	0.2314	0.1781	0.1735	0.1765	0.1781
Money	Insur	0.1104	0.1417	0.1584	0.0839	−0.0180	−0.0752	−0.0715
Money	RlEst	0.7355	0.7525	0.7310	0.7134	0.6523	0.7150	0.8606
Other	BldMt	0.3607	0.3596	0.3574	0.3410	0.3269	0.3968	0.2694
Other	BusSv	0.4879	0.5111	0.4951	0.4573	0.4539	0.4040	0.4887
Other	Cnstr	0.6069	0.5831	0.6786	0.5967	0.6344	0.5671	0.5042
Other	Fun	0.2688	0.2297	0.2801	0.2886	0.2084	0.4270	0.5197
Other	Gold	0.4373	0.3336	0.3802	0.6402	0.5836	0.2535	0.7318
Other	Meals	0.1580	0.1199	0.1692	0.1784	0.1175	0.2247	0.2498
Other	Other	0.3038	0.3289	0.3575	0.2765	0.3156	0.1769	0.2268
Other	Trans	0.2698	0.2699	0.2786	0.2747	0.2905	0.3822	0.2701
Shops	PerSv	0.5884	0.6341	0.6142	0.4737	0.4157	0.4531	0.7126
Shops	Rtail	0.0531	0.0586	0.0721	0.0506	0.0031	0.0346	0.0209
Shops	Whlsl	0.4161	0.4150	0.3876	0.4291	0.3849	0.4720	0.4194
Telecommunications	Telcm	−0.1961	−0.1676	−0.2229	−0.2066	−0.2111	−0.2495	−0.0907
Utilities	Util	−0.0315	0.0290	−0.0185	−0.0309	−0.1474	−0.2308	−0.3373

Table 17. Average parameters for SMB by industry.

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Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	Chips	2.0270	1.6195	1.5146	2.0304	2.8404	3.9464	4.8717
Business Equipment	Hardw	−2.1448	−3.1135	−2.2787	−1.4227	−0.1158	1.1290	1.8938
Business Equipment	Softw	2.4408	2.2554	2.7149	2.4298	2.6073	3.4493	5.0468
Chemicals	Chems	0.8049	0.4926	1.0285	1.1941	0.9490	0.3295	0.7415
Consumer Durables	Hshld	−3.5220	−3.3883	−3.5238	−3.5921	−2.8983	−2.1829	−4.8518
Consumer Non-Durables	Agric	2.5105	2.2923	2.1719	2.8455	2.6389	3.4121	5.8980
Consumer Non-Durables	Beer	−1.7689	−1.6097	−1.7492	−2.0344	−2.1755	−0.8634	−2.7206
Consumer Non-Durables	Books	3.2699	3.0658	3.0600	3.0247	2.8598	2.3327	2.7036
Consumer Non-Durables	Clths	4.6459	4.2592	4.4068	4.4906	4.3727	4.9556	6.6808
Consumer Non-Durables	Food	−0.7780	−0.2173	−0.7536	−0.6514	−1.4482	−1.7753	−2.1650
Consumer Non-Durables	Smoke	−1.9182	−1.6316	−1.6903	−2.2337	−1.8367	−1.8603	−3.0293
Consumer Non-Durables	Soda	−1.2139	−0.7835	−1.2947	−1.6556	−2.0505	−0.5054	−1.6006
Consumer Non-Durables	Toys	2.2761	1.8055	2.1091	2.3271	3.2539	5.5906	2.8274
Consumer Non-Durables	Txtls	5.8602	5.3274	5.3666	5.8874	6.9126	6.0350	7.7287
Energy	Coal	2.1650	2.0755	1.9526	2.3896	3.6066	2.7436	2.3730
Energy	Mines	3.1107	2.7770	3.0812	3.2862	2.9662	2.7582	5.6042
Energy	Oil	−2.0905	−2.3782	−1.7681	−1.6474	−1.0455	−0.8614	−0.0473
Health	Drugs	−2.4303	−1.6248	−2.5461	−2.7498	−2.7144	−3.3888	−4.1032
Health	Hlth	4.0472	3.8694	3.9082	3.8369	3.8314	4.3567	6.1027
Health	MedEq	1.0414	1.1860	1.1565	0.5778	0.6632	1.1281	2.7113
Manufacturing	Aero	0.7459	0.2913	0.6606	1.1866	1.6213	1.3320	2.0286
Manufacturing	Autos	0.0875	−0.5022	0.1685	0.8437	0.8694	0.8037	3.2068
Manufacturing	Boxes	0.9961	1.2750	0.9464	1.0394	0.8415	−0.1866	−2.0069
Manufacturing	ElcEq	1.3074	1.1908	0.7994	1.6185	2.1009	2.3364	1.8252
Manufacturing	FabPr	5.1538	4.7242	4.9314	5.4761	6.3364	6.1554	7.7016
Manufacturing	Guns	−0.1197	−0.4346	−0.4361	0.5318	0.2840	0.5103	0.2687
Manufacturing	LabEq	3.4862	3.0238	3.1742	3.5787	3.7878	5.3567	8.5789
Manufacturing	Mach	4.9237	4.6258	4.3522	4.9186	5.8787	5.6268	5.8431
Manufacturing	Paper	1.3213	1.5565	1.2804	1.0202	0.5431	0.0406	0.6256
Manufacturing	Rubbr	5.4436	4.8395	4.8241	5.9850	6.1595	7.2427	7.7446
Manufacturing	Ships	2.2170	2.0087	2.0331	2.6819	2.0796	2.0421	3.0639
Manufacturing	Steel	3.8506	3.5880	4.0202	3.9397	4.2276	3.9885	6.3386
Money	Banks	0.8555	1.1092	0.5180	0.2203	0.4598	−1.4619	−0.9630
Money	Fin	4.0807	3.8641	3.9287	3.5456	3.3965	2.9959	2.9406
Money	Insur	1.5832	1.9682	2.0359	1.1199	−0.1656	−0.8606	−1.7632

Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Money	RlEst	5.9944	5.4956	5.7325	5.9163	6.1071	7.1971	10.0976
Other	BldMt	5.3071	5.0776	5.0110	4.7677	4.9501	5.8093	4.5240
Other	BusSv	8.6517	8.1784	8.3038	8.4158	9.0863	9.5592	13.4606
Other	Cnstr	5.0753	4.4826	5.3446	4.8955	5.8743	6.6856	6.7519
Other	Fun	1.9042	1.6188	1.9848	2.1492	1.5732	3.7308	4.8763
Other	Gold	1.4068	0.9538	1.1572	2.0191	2.3464	1.1064	2.9292
Other	Meals	1.5560	1.3460	1.6073	1.5225	0.8809	2.6134	2.4165
Other	Other	3.6119	3.2794	3.6755	3.2138	4.4955	3.2650	4.9368
Other	Trans	3.0650	2.9089	3.0021	3.0266	3.4103	4.8166	3.8428
Shops	PerSv	5.0862	5.1274	4.9562	4.2433	3.8334	4.6278	9.3979
Shops	Rtail	0.8020	0.8529	0.9481	0.7217	0.0397	0.3771	−0.9400
Shops	Whlsl	6.3787	5.7385	5.6478	6.6311	7.2358	10.2466	10.6375
Telecommunications	Telcm	−2.4523	−1.9666	−2.4536	−2.4007	−3.0064	−3.6550	−1.8704
Utilities	Util	−0.2203	1.1265	0.0408	−0.2535	−1.9483	−3.0557	−5.3771

Table 18. Average t-statistics for the size parameter, SMB.

Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	Chips	−0.5377	−0.5285	−0.5752	−0.6469	−0.5214	−0.4729	−0.3322
Business Equipment	Hardw	−0.8098	−0.8607	−0.8808	−0.8076	−0.7578	−0.7139	−0.6310
Business Equipment	Softw	−0.6364	−0.5303	−0.6027	−0.6813	−0.7367	−0.7481	−0.7859
Chemicals	Chems	0.1927	0.1449	0.1889	0.2278	0.2695	0.2417	0.3472
Consumer Durables	Hshld	−0.2313	−0.2352	−0.2307	−0.2223	−0.2604	−0.3878	−0.2267
Consumer Non-Durables	Agric	0.0095	−0.0165	−0.0158	0.0992	0.0344	−0.0904	0.0845
Consumer Non-Durables	Beer	−0.2143	−0.2016	−0.1889	−0.2460	−0.2633	−0.2624	−0.0864
Consumer Non-Durables	Books	0.0862	0.0981	0.0933	0.0987	0.0568	0.0769	0.2010
Consumer Non-Durables	Clths	0.1122	0.1421	0.1134	0.0999	0.1115	−0.0405	−0.0016
Consumer Non-Durables	Food	−0.0502	−0.0040	−0.0396	−0.1035	−0.1418	−0.0782	−0.0692
Consumer Non-Durables	Smoke	−0.1436	−0.1884	−0.1401	−0.1125	−0.0478	−0.0634	−0.0241
Consumer Non-Durables	Soda	−0.1529	−0.1422	−0.1075	−0.2756	−0.2496	−0.2288	−0.0798
Consumer Non-Durables	Toys	−0.1494	−0.1311	−0.1945	−0.2309	−0.0376	−0.0763	−0.1821
Consumer Non-Durables	Txtls	0.3158	0.3260	0.3353	0.3410	0.1587	0.1391	0.4091
Energy	Coal	0.5011	0.3591	0.5568	0.5143	0.6019	0.8063	0.8292
Energy	Mines	0.3804	0.3116	0.4216	0.4245	0.5255	0.3970	0.5823
Energy	Oil	0.4514	0.3939	0.4899	0.5467	0.5338	0.5298	0.5488

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Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Health	Drugs	−0.5554	−0.5052	−0.5491	−0.5895	−0.5725	−0.6042	−0.7282
Health	Hlth	−0.1292	−0.0484	−0.1273	−0.1609	−0.2080	−0.1178	−0.2578
Health	MedEq	−0.3412	−0.2495	−0.3072	−0.4346	−0.4019	−0.5114	−0.4813
Manufacturing	Aero	0.1564	0.1499	0.1667	0.1944	0.2285	0.0375	0.1123
Manufacturing	Autos	0.5113	0.5198	0.4965	0.5262	0.5037	0.4832	0.5802
Manufacturing	Boxes	0.0993	0.1472	0.0730	0.0643	0.1148	0.0842	−0.1027
Manufacturing	ElcEq	−0.0827	−0.0828	−0.1275	−0.0159	−0.1266	−0.2090	−0.1646
Manufacturing	FabPr	0.2800	0.3523	0.2280	0.2061	0.2466	0.1813	0.1398
Manufacturing	Guns	0.1831	0.2354	0.1139	0.1712	0.1130	0.0979	0.2811
Manufacturing	LabEq	−0.3127	−0.2836	−0.2694	−0.3621	−0.3496	−0.4837	−0.5052
Manufacturing	Mach	0.1358	0.1126	0.1292	0.1661	0.1470	0.1298	0.2192
Manufacturing	Paper	0.2291	0.2275	0.2272	0.2515	0.2642	0.1278	0.3562
Manufacturing	Rubbr	0.2029	0.2476	0.2163	0.1876	0.1510	0.0427	0.0903
Manufacturing	Ships	0.2085	0.2072	0.2072	0.2523	0.1929	0.1642	0.0738
Manufacturing	Steel	0.6477	0.5966	0.6558	0.7183	0.7811	0.5844	0.7460
Money	Banks	0.5708	0.5330	0.5573	0.5681	0.5618	0.6051	0.6895
Money	Fin	0.3545	0.3418	0.3303	0.3266	0.3530	0.3187	0.3266
Money	Insur	0.3219	0.3246	0.3257	0.2904	0.3173	0.3464	0.3412
Money	RlEst	0.3227	0.3210	0.3064	0.2561	0.2363	0.3058	0.4090
Other	BldMt	0.2165	0.1679	0.2457	0.2584	0.2389	0.1483	0.2445
Other	BusSv	−0.0287	0.0044	−0.0173	−0.0309	−0.0591	−0.0452	−0.1486
Other	Cnstr	0.3317	0.3605	0.3720	0.2881	0.2687	0.4498	0.5322
Other	Fun	−0.1184	−0.0801	−0.1530	−0.1116	−0.1949	−0.2539	−0.0978
Other	Gold	0.1358	0.0125	0.0852	0.2177	0.2851	0.2729	0.7532
Other	Meals	−0.2193	−0.2227	−0.2296	−0.2285	−0.2175	−0.3029	−0.2790
Other	Other	0.0386	0.0365	0.0558	0.1012	0.0031	0.0910	0.0652
Other	Trans	0.2608	0.2783	0.2557	0.2630	0.2673	0.2069	0.1664
Shops	PerSv	0.0106	0.0754	0.0059	−0.0664	0.0108	−0.0061	−0.0195
Shops	Rtail	−0.1028	−0.0794	−0.0716	−0.1005	−0.1233	−0.2047	−0.3034
Shops	Whlsl	0.0510	0.0798	0.0577	0.0503	0.0075	−0.0043	−0.0177
Telecommunications	Telcm	0.2525	0.3053	0.2749	0.2256	0.1955	0.2742	0.0969
Utilities	Util	0.4179	0.3718	0.4217	0.4349	0.4668	0.5126	0.3949

Table 19. Average parameters for HML by industry.

Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Business Equipment	Chips	−3.8316	−3.6320	−3.7992	−4.6833	−4.4293	−4.9713	−3.3804
Business Equipment	Hardw	−4.8485	−4.7393	−5.0597	−5.2962	−6.1684	−6.4471	−5.7814
Business Equipment	Softw	−4.9513	−4.2904	−4.7301	−5.5315	−6.4523	−6.0185	−8.9519
Chemicals	Chems	1.5807	1.0305	1.5022	2.0940	2.7400	2.9636	4.0330
Consumer Durables	Hshld	−2.3809	−2.0684	−2.3437	−2.4775	−3.7354	−5.6384	−4.9896
Consumer Non-Durables	Agric	0.0740	−0.1157	0.0024	0.4892	0.3488	−0.5291	−0.0022
Consumer Non-Durables	Beer	−1.5311	−1.3189	−1.5000	−1.8565	−2.1101	−2.3938	−1.5355
Consumer Non-Durables	Books	0.9144	0.9247	0.9670	1.1610	0.5773	1.1321	2.4769
Consumer Non-Durables	Clths	0.9467	0.9735	0.9728	0.9999	1.2157	0.3739	0.5315
Consumer Non-Durables	Food	−0.4112	0.0323	−0.3587	−0.8661	−1.6998	−0.7677	−0.8502
Consumer Non-Durables	Smoke	−0.9420	−1.0478	−0.9154	−0.9083	−0.7504	−0.5228	−0.6924
Consumer Non-Durables	Soda	−0.8866	−0.7657	−0.7021	−1.2258	−1.5206	−1.9345	−2.9555
Consumer Non-Durables	Toys	−0.6113	−0.3523	−0.7208	−1.2232	−0.2576	−0.4538	−1.7159
Consumer Non-Durables	Txtls	2.3341	2.0792	2.3535	2.7470	1.8222	2.0718	4.8854
Energy	Coal	1.4393	0.9462	1.5578	1.7379	2.4123	2.6359	3.6478
Energy	Mines	2.0426	1.5405	2.2404	2.5477	3.3198	2.5847	3.8822
Energy	Oil	2.6581	2.3133	2.7992	3.2025	3.9142	4.3390	4.3149
Health	Drugs	−5.0907	−4.3717	−4.8950	−5.6002	−6.1844	−6.6855	−8.9212
Health	Hlth	−0.6160	−0.1308	−0.6923	−0.9001	−1.3981	−0.7229	−2.5513
Health	MedEq	−2.7898	−1.9996	−2.4451	−3.7228	−4.1971	−5.7059	−5.6987
Manufacturing	Aero	0.9648	0.6773	1.0958	1.5436	1.9928	0.6222	1.5235
Manufacturing	Autos	3.2835	2.9080	3.1879	3.9504	4.1288	4.5340	5.8731
Manufacturing	Boxes	0.6010	0.7789	0.5898	0.4788	0.7280	0.3423	−2.5781
Manufacturing	ElcEq	−0.3872	−0.3341	−0.4123	0.0537	−1.0260	−2.1100	−1.3577
Manufacturing	FabPr	1.4724	1.6384	1.1517	1.2185	1.6245	1.3601	1.6359
Manufacturing	Guns	0.6623	0.7106	0.3766	0.9148	0.6122	0.6942	1.9906
Manufacturing	LabEq	−2.2409	−1.9613	−1.9003	−2.3894	−3.1570	−4.9967	−6.9546
Manufacturing	Mach	1.4261	1.1052	1.5513	1.7953	1.6190	1.2927	2.5047
Manufacturing	Paper	1.8917	1.6192	1.9279	2.3544	2.4226	1.2835	4.0063
Manufacturing	Rubbr	1.5629	1.6763	1.5835	1.5672	1.4813	0.5814	1.0477
Manufacturing	Ships	1.1188	1.0080	1.1144	1.4306	1.5571	1.2472	0.2335
Manufacturing	Steel	4.4322	3.7856	4.3600	5.3271	6.1578	5.2726	6.4820
Money	Banks	6.5329	6.1530	6.1435	6.6598	6.7497	7.9405	10.1990
Money	Fin	4.4794	3.9067	4.2282	4.8100	5.5739	5.1944	6.3825
Money	Insur	3.8657	3.7464	3.8372	3.8679	4.0888	4.6434	5.1487
Money	RlEst	2.2449	2.0399	2.1946	1.8982	2.0844	2.7702	5.0913

Sector	Industry	Standard	Scale 1	Scale 2	Scale 3	Scale 4	Scale 5	Scale 6
Other	BldMt	2.3264	1.7688	2.4867	2.8982	2.8896	2.1000	3.7157
Other	BusSv	−0.5466	−0.1084	−0.4198	−0.5681	−1.3059	−1.3618	−4.3800
Other	Cnstr	2.1102	2.0812	2.2255	1.9296	1.9579	4.0280	5.2924
Other	Fun	−0.5212	−0.2189	−0.7152	−0.4888	−1.3764	−1.9931	−1.2353
Other	Gold	0.1164	−0.1277	−0.0720	0.3751	0.7530	0.3273	1.9220
Other	Meals	−1.5515	−1.3257	−1.5194	−1.7373	−2.2743	−3.2196	−3.3617
Other	Other	0.4663	0.5129	0.5866	0.7305	0.0857	1.0849	2.0143
Other	Trans	2.1981	2.0402	2.1532	2.5688	2.7935	2.5544	1.9368
Shops	PerSv	0.2821	0.5863	0.2316	−0.1287	0.2114	0.1905	−0.2324
Shops	Rtail	−1.2744	−1.0379	−0.9744	−1.3434	−1.8573	−2.4995	−3.1044
Shops	Whlsl	0.3727	0.5534	0.4743	0.4695	−0.2664	−0.2654	−1.1328
Telecommunications	Telcm	2.0616	2.0806	2.1254	1.9210	2.2603	3.3976	2.7911
Utilities	Util	4.8274	4.3822	4.9051	5.3088	5.5951	6.5609	5.8998

Table 20. Average t-statistics for the book-to-market parameter, HML.

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